A stochastic feedback system model of a stock exchange

László Gerencsér
Zalán Mátyás*

Stochastic Systems Research Group,
MTA SZTAKI

Morgan Stanley's Quantitative and
Mathematical Finance Conference

October 20-21, 2005
Alfréd Rényi Institute of Mathematics,
Budapest, Hungary

Outline

- Agent-based market models

- A behavioral stock market model

- Updating the beliefs

- Market equilibrium

- Simulation results

Stock prices

Traditional modelling: "exogenous" evolution

$$dS_t = S_t\,(\mu dt + \sigma dW_t)$$

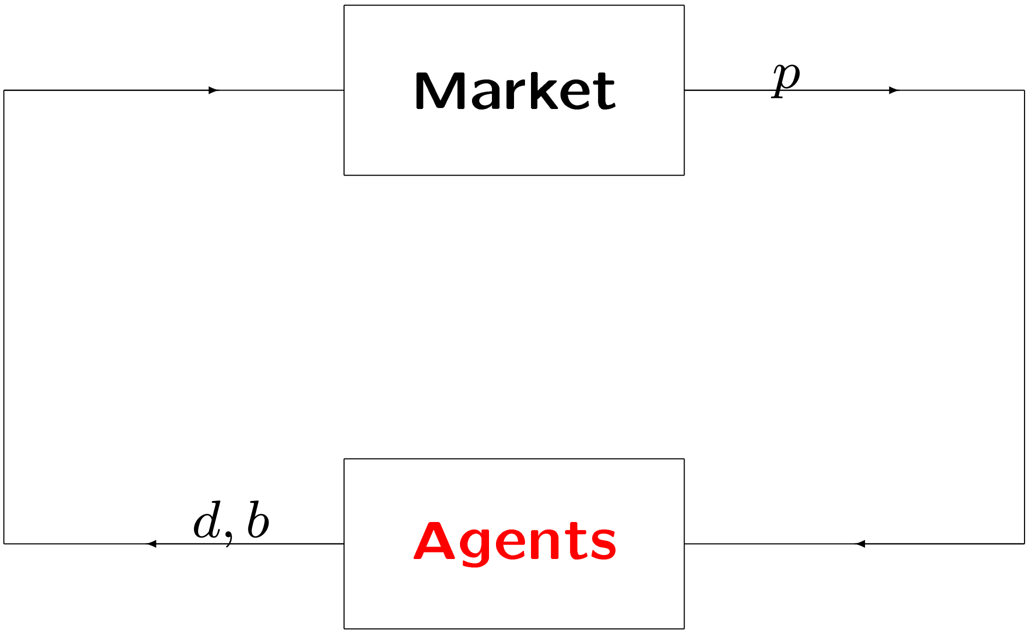
Agent-based modelling:

O'Hara: Market Microstructure Theory, 2000

Santa Fe Artificial Stock Market, 1997

Closed-loop market models

Mutual causality between demand and price $\longrightarrow$ **closed-loop modeling**



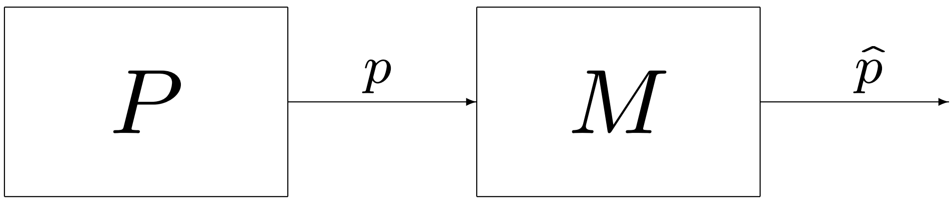
Market: known regulations (e.g. NYSE rules)

Agents: transaction requests based on beliefs and behavior

Agent's beliefs

The one-step ahead price predictor is:

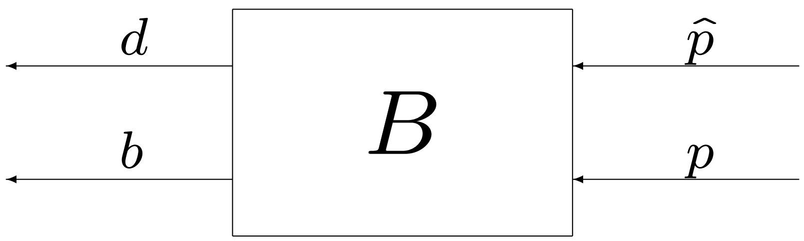
$$\widehat{p} = Mp.$$



Agent's behavior

Compute a transaction request according to the *behavior* B:

$$(d, b) = B(p, \widehat{p}).$$



Behavioral finance:

Kahnemann and Tversky, Econometrica, 1979

Kostolany, Börsenpsyhologie, 1991

Example – demand

Loss aversion (Kahnemann and Tversky):

$$\boxed{1 \text{ \$ loss} = 2.5 \text{ \$ gain}}$$

Risk aversion is severely biased!

A possible realization:

$$d_n = \begin{cases} +1\,|\widehat{p}_n - p_{n-1}| & \text{if } \widehat{p}_n > p_{n-1}, \\ -0.4\,|\widehat{p}_n - p_{n-1}| & \text{if } \widehat{p}_n \leq p_{n-1}. \end{cases}$$

Example – bid price

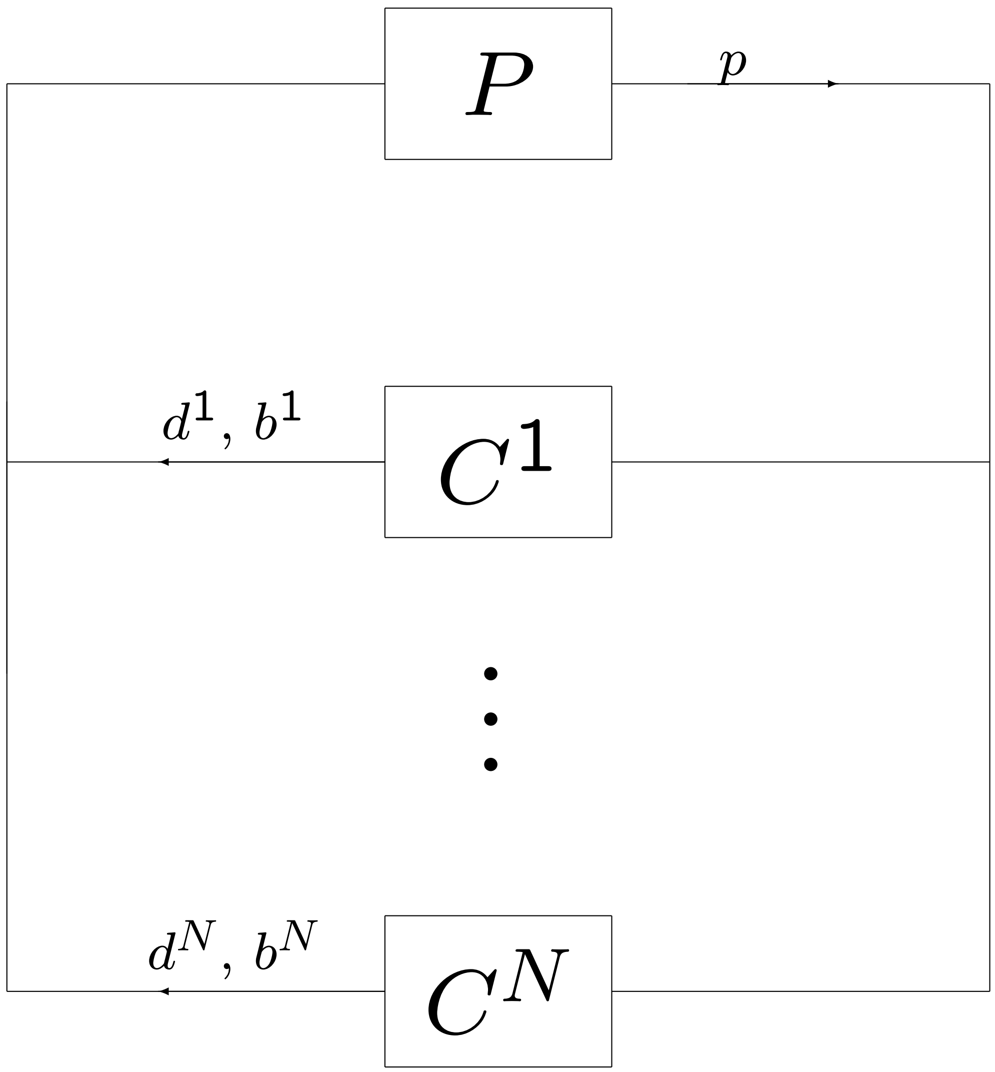
Markup bid prices:

$$b_n = \begin{cases} p_{n-1}(1-\varepsilon) & \text{if selling} \\ p_{n-1}(1+\varepsilon) & \text{if buying.} \end{cases}$$

Mean rule:

$$b_n = \frac{\widehat{p}_n + p_{n-1}}{2}$$

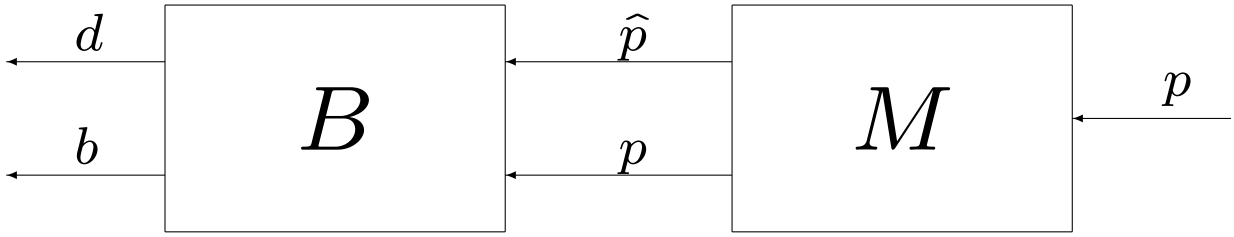
A system theoretic view



P: plant = stock market rules

C: controller = agent

The controller



Controller = Predictor + **Behavior**

Introducing randomness

- *Threshold* dependent action

$$(d,b) = \begin{cases} B(p,\widehat{p}) & \text{if } |\widehat{p} - p| > 0.01p \\ Z & \text{otherwise,} \end{cases}$$

where Z is a random variable.

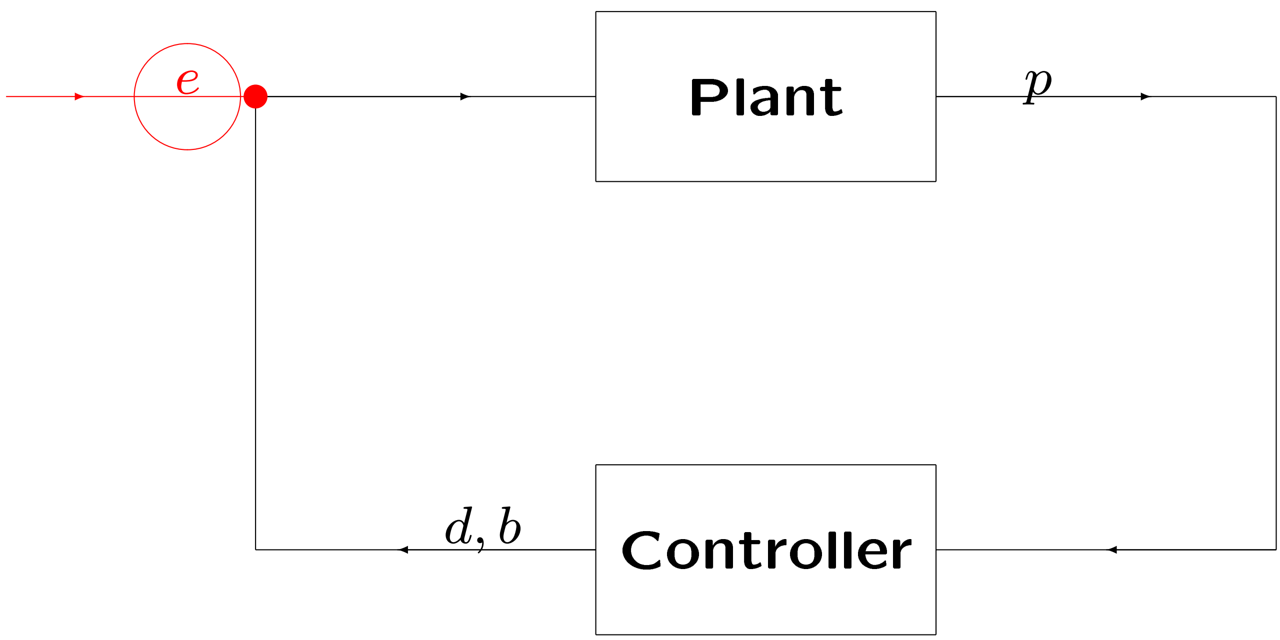
- Limited *confidence*

$$(d,b) = \begin{cases} B(p,\widehat{p}) & \text{with } 90\% \\ Z & \text{with } 10\%. \end{cases}$$

Exogenous noise

Random adjustment of transaction requests:

$$(d, b) = B(p, \hat{p}) + e$$



$AR(k)$ predictors for p

Approximate p by a low order AR process. Let

$$A(\theta) = 1 + \theta_1 q^{-1} + \ldots + \theta_k q^{-k}$$

and

$$D_k = \{\theta \in \mathbb{R}^k \mid g(z) = 1+\theta_1 z+\ldots+\theta_k z^k \text{ is stable}\}.$$

Solve the *approximation problem*

$$E|A(\theta)p|^2 \to \min_{\theta \in D_k}!$$

Let θ^* denote the solution. The best $AR(k)$ predictor of p is given by

$$\widehat{p} = Mp$$

where

$$M = M(\theta^*) = 1 - A(\theta^*).$$

The adjustment mechanism

1. An *a priori belief*: assume a model $\eta \in D_k$ for p.

2. The agent's *action*: use the best $AR(k)$ predictor

$$M(\eta) = I - A(\eta).$$

A closed loop price process $p(\eta)$ emerges.

3. *Adjusting* the predictor: fit an $AR(k)$ process to $p(\eta)$ using LSQ.

Denote the parameter of the new belief by

$$\varphi(\eta).$$

Adjusting the predictor

For an assumed η and a running $\theta \in D_k$ define the estimated *innovation process*

$$\nu(\theta, \eta) := A(\theta) p(\eta).$$

The approximation problem: minimize

$$W(\theta, \eta) := \frac{1}{2} E \, |\nu(\theta, \eta)|^2$$

subject to $\theta \in D_k$, i.e. solve

$$W_\theta(\theta, \eta) = 0.$$

Remark. Note that the *gradient* w.r.t. θ is

$$\nu_\theta(\theta, \eta) = (p_{n-1}(\eta), \ldots, p_{n-k}(\eta))^T.$$

Market equilibrium

Problem: find the market equilibrium, i.e.

$$\varphi(\eta) = \eta.$$

Equivalently: find η such that

$$W_\theta(\eta, \eta) = E\nu_\theta(\eta, \eta)\nu(\eta, \eta) = 0.$$

This is non-linear in η!

A data-driven procedure

A stochastic gradient procedure:

$$\eta_{n+1} = \eta_n - \frac{1}{n}\, \nu_{\theta n} \nu_n.$$

Here ν_n and $\nu_{\theta n}$ are on-line estimates defined by

$$\nu_n = [A(\eta_n)p]_n$$

$$\nu_{\theta n} = (p_{n-1}, \ldots, p_{n-k})$$

where (p_n) is the *observed* price process.

Application of BMP theory

The price dynamics is *non-linear* but Markovian in terms of the state vector

$$X_n = (p_{n-1}, \ldots, p_{n-k})^T.$$

Reference:

Benveniste, Métivier, Priouret: Adaptive Algorithms and Stochastic Approximations, 1990

Provides *sufficient conditions* for convergence.

Basic conditions of BMP theory

A basic tool: geometric ergodicity

For fixed η denote the transition kernel by Π_η.
A key condition:

Exponential stability w.r.t. initial conditions:

$$|\Pi_\eta^k g(x) - \Pi_\eta^k g(x')| \leq C_g\, \rho^k |x - x'|(1 + |x|^p + |x'|^p)$$

where $g \in Li(p)$ and $0 < \rho < 1$.

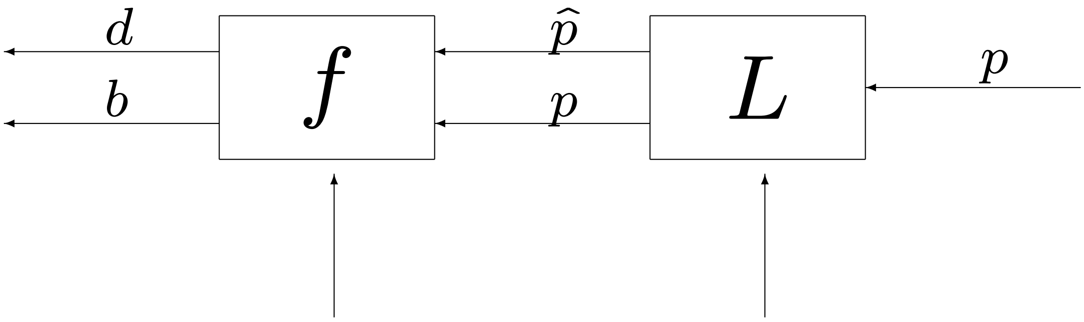
Reference:

Gerencsér and Mátyás: A.s. convergence with resetting, 2005

Convergence of the SA-procedure

Market: static nonlinearity

Agents: Wiener-structure



static nonlinearity   linear filter

f: linear growth and Lipschitz condition

Price predictor: 'close' to $AR(1)$

A simulation result

Market: opening rules at BÉT

Agents:

Agent 1: loss aversion

Agent 2: rational behavior

Agent 3: risk seeking

Agent 4: noise trader

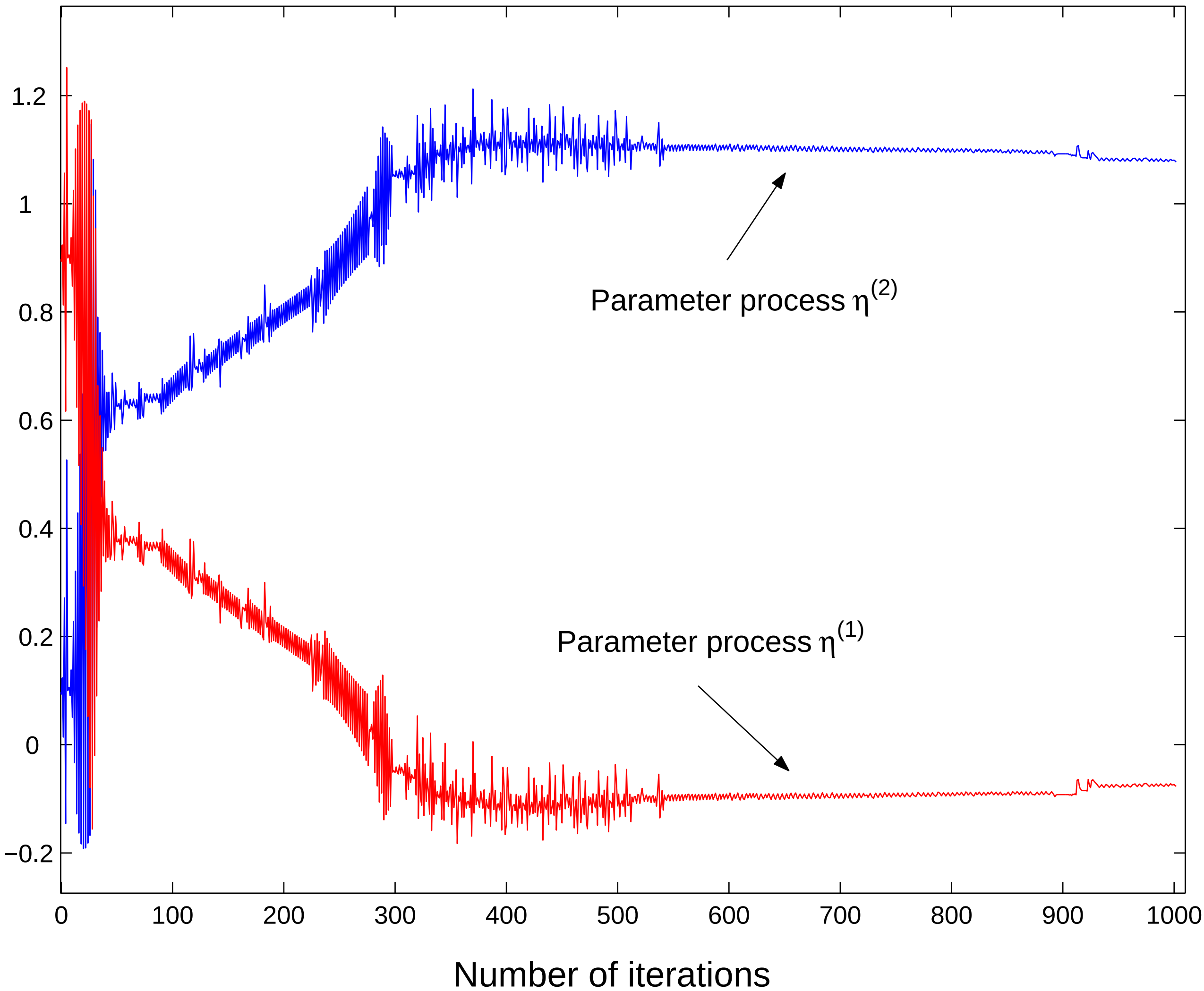
Bid prices: markup

Randomness: threshold dependent actions

The parameter process

$AR(2)$ predictor:

$$\widehat{p}_t = \eta_t^{(1)} p_{t-1} + \eta_t^{(2)} p_{t-2}$$



Conclusion

- A behavioral stock market model has been proposed

- An adjustment mechanism has been examined

- Reaching market equilibrium: SA-procedure analyzed using BMP theory

- A simulation result has been presented

Main references

[1] L. Gerencsér, Z. Mátyás: *A system theoretic approach to behavioral finance*, 43rd IEEE Conference on Decision and Control, 2004.

[2] D. Kahneman, A. Tversky, *Prospect theory: An analysis of decision under risk*, Econometrica, vol. 47, 1979, pp. 263-292.

[3] A. Benveniste, M. Métivier, P. Priouret, *Adaptive Algorithms and Stochastic Approximations*, Springer Verlag, Berlin, 1990.